

FRACTIONS MODULE

Part I

I. Basics of Fractions

II. Rewriting Fractions in the Lowest Terms

III. Change an Improper Fraction into a Mixed Number

IV. Change a Mixed Number into an Improper Fraction

I. Fraction: Basics

Introduction: This is the first of four parts on working with fractions. You're going to review a few definitions regarding fractions. Afterwards, you're going to try to do some problems on your own. There will be 20 problems for you to practice. After you're successful in doing the practice problems, try the short quiz. The answers can be found at the end of each section.

Definitions:

The numerator is written above the fraction bar and the denominator is written under the fraction bar.

$$\frac{\text{Numerator}}{\text{Denominator}}$$

Another way to look at defining a fraction is written as a part of the whole. The part is always written in the numerator. The whole is always written in the denominator.

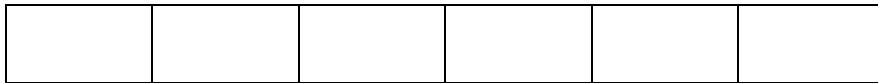
$$\frac{\text{Part}}{\text{Whole}}$$

A) Example: What part of the bar is shaded?



Step 1: Count how many pieces the bar contains.

The whole bar is broken up into 6 pieces.



Therefore the whole = 6 pieces.

Step 2: Count how many pieces of the bar is shaded.

The part of the bar that is shaded is 5 pieces.



Therefore, the part = 5 pieces.

Step 3: Put all the information from Step 1 and Step 2 together to get a fraction.

The numerator is 5 and the denominator is 6.

$$\frac{\text{Part}}{\text{Whole}}$$

Answer: $\frac{5}{6}$ of the bar is shaded

B) Example: What part of the bar is not shaded?



Step 1: The bar has how many pieces? 6

This is the denominator or the whole.

Step 2: The bar has how many pieces not shaded? 1

This is the numerator or the part.

Step 3: Write the information from Step 1 and Step 2 in fraction format.

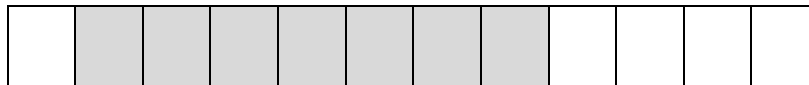
$$\frac{1}{6}$$

Answer: $\frac{1}{6}$ of the bar is not shaded.

C) Example:

a) **What part of the bar is shaded?**

b) **What part of the bar is not shaded?**



a) **What part of the bar is shaded?**

Step 1: 12 pieces

Step 2: 7 pieces are shaded

Step 3: $\frac{7}{12}$ of the bar is shaded.

b) **What part of the bar is not shaded?**

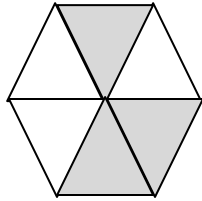
Step 1: 12 pieces

Step 2: 5 pieces are not shaded

Step 3: $\frac{5}{12}$ of the bar is not shaded.

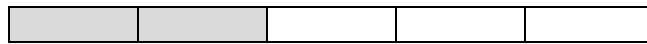
D) You Try:

1. In the figure:



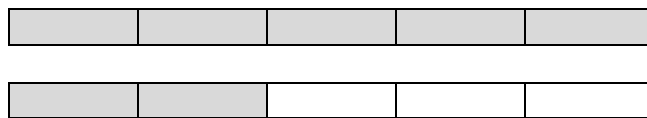
- a. What part of the figure is shaded? _____
- b. What part of the figure is not shaded? _____

2. In the figure:



- a. What part of the figure is shaded? _____
- b. What part of the figure is not shaded? _____

3. BONUS. In the figure:



- a. What part of the figure is shaded? _____
- b. What part of the figure is not shaded? _____

Answers to D "You Try" : 1a. $\frac{3}{6}$ or $\frac{1}{2}$, 1b. $\frac{3}{6}$ or $\frac{1}{2}$, 2a. $\frac{2}{5}$, 2b. $\frac{3}{5}$, 3a. $1\frac{2}{5}$, 3b. $\frac{3}{5}$

E) Identifying the Types of Fractions.

Three Types of Fractions

1. Proper Fraction
2. Improper Fraction
3. Mixed Number

1) Proper Fraction

Definition: A proper fraction is a fraction with a numerator that is smaller than the denominator.

Examples: $\frac{3}{4}$, $\frac{5}{12}$, $\frac{7}{213}$, $\frac{6}{7}$, $\frac{1}{2}$

2) Improper Fraction

Definition: An improper fraction is a fraction with a numerator that is larger than the denominator.

Examples: $\frac{4}{3}$, $\frac{12}{5}$, $\frac{213}{7}$, $\frac{7}{6}$, $\frac{2}{1}$

3) Mixed Number

Definition: A number that is made up of the sum of a whole number and a proper fraction.

$$1 + \frac{4}{5} = 1\frac{4}{5}$$

Examples: $1\frac{1}{3}$, $2\frac{3}{5}$, $30\frac{3}{7}$, 2

F) PRACTICE PROBLEMS. (20)

Write a fraction that represents the shaded part of the figure.

1)



2)



3)



4)



5)



Identify what type of fraction for the following:

6) $\frac{3}{4}$

7) $1\frac{2}{5}$

8) $\frac{4}{5}$

9) $\frac{7}{3}$

10) $\frac{5}{1}$

11) $1\frac{3}{7}$

12) $\frac{6}{7}$

13) $\frac{9}{5}$

14) $2\frac{3}{8}$

15) $3\frac{4}{7}$

16) $\frac{1}{2}$

17) $\frac{1}{4}$

18) $1\frac{1}{2}$

19) $1\frac{3}{4}$

20) $\frac{13}{4}$

Answers to F “Practice Problems”: 1) $\frac{3}{5}$; 2) $\frac{1}{4}$; 3) $\frac{4}{6} = \frac{2}{3}$; 4) $\frac{1}{2}$; 5) $\frac{2}{2} = 1$; 6) proper fraction; 7) mixed number; 8) proper fraction; 9) improper fraction; 10) improper fraction; 11) mixed number; 12) proper fraction; 13) improper fraction ; 14) mixed number; 15) mixed number; 16) proper fraction; 17) proper fraction; 18) mixed number; 19) mixed number; 20) improper fraction

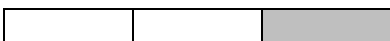
G) QUIZ. (5 questions)

Write a fraction that represents the shaded part of the figure.

1)



2)



Identify the type of fraction for the following:

3) $\frac{3}{2}$

4) $\frac{2}{3}$

5) $1\frac{2}{3}$

Answers to G “Quiz”: 1) $\frac{2}{4} = \frac{1}{2}$; 2) $\frac{1}{3}$; 3) improper fraction; 4) proper fraction; 5) mixed number

II. Rewriting Fractions in Lowest Terms.

This is the second of four parts on understanding fractions. You're going to look at examples on how to rewrite fractions in lowest terms. Afterwards, you're going to try to do some problems on your own. There will be 20 problems for you to practice. After you're successful in doing the practice problems, try the short quiz. The answers can be found at the end of each section.

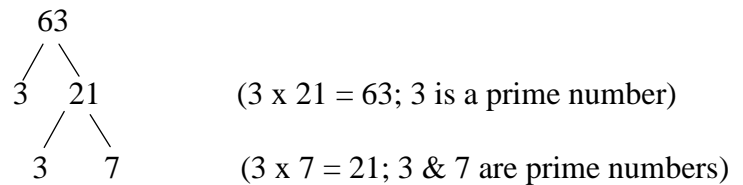
A) Method 1: Prime Factorization

Example: Rewrite $\frac{63}{81}$ in lowest terms.

STEP 1: Find the prime factorization for 63.

Recall that **prime numbers** are numbers that can only be divided by the number and one. The number 1 is not a prime number. Here is a list of a first ten prime numbers: **2, 3, 5, 7, 11, 13, 17, 19, 23, 29.**

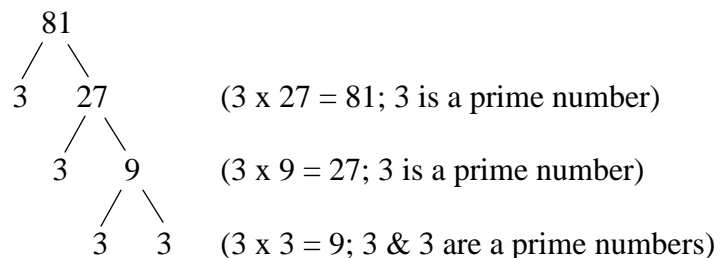
Use a prime factorization tree to break down 63 into its prime numbers.



The prime factorization of **63 = 3 x 3 x 7.**

STEP 2: Find the prime factorization for 81

Use a prime factorization tree to break down 81 into its prime numbers.



The prime factorization for **81 = 3 x 3 x 3 x 3**

STEP 3: Rewrite the fraction using the prime factorizations of both 63 and 81.

$$\frac{63}{81}$$

$$\frac{3 \times 3 \times 7}{3 \times 3 \times 3 \times 3}$$

STEP 4: Cancel like factors.

$$\frac{\cancel{3}x\cancel{3}x7}{\cancel{3}x\cancel{3}x3x3}$$

STEP 5: Multiply the left over factors to get your new fraction reduced to lowest terms.

$$\frac{7}{9}$$

B) Method 2: Guess and Reduce

Example: Rewrite $\frac{24}{54}$ in lowest terms.

STEP 1: Guess what number can be divided into the numerator and then the denominator without a remainder.

Since both numbers are even, guess 2.

STEP 2: Divide numerator and denominator by 2.

$$\frac{24 \div 2}{54 \div 2}$$

$$\frac{12}{27}$$

STEP 3: Guess what number can be divided into the numerator and then the denominator of the fraction in STEP 2 without a remainder.

Try dividing each number by 3.

$$\frac{12 \div 3}{27 \div 3}$$

$$\frac{4}{9}$$

C) You Try:

a) $\frac{25}{35}$

b) $\frac{24}{64}$

c) $\frac{13}{52}$

Answers to C “You Try”:

a) $\frac{5}{7}$

b) $\frac{3}{8}$

c) $\frac{1}{4}$

D) PRACTICE PROBLEMS.

Rewrite the following fractions in lowest terms.

1) $\frac{4}{8}$

2) $\frac{3}{9}$

3) $\frac{2}{10}$

4) $\frac{2}{14}$

5) $\frac{5}{15}$

6) $\frac{16}{24}$

7) $\frac{12}{36}$

8) $\frac{49}{63}$

9) $\frac{18}{42}$

10) $\frac{17}{34}$

11) $\frac{26}{52}$

12) $\frac{3}{18}$

13) $\frac{17}{34}$

14) $\frac{22}{77}$

15) $\frac{15}{33}$

16) $\frac{27}{45}$

17) $\frac{4}{78}$

18) $\frac{34}{78}$

19) $\frac{28}{68}$

20) $\frac{20}{200}$

Answers to D "Practice Problems": 1) $\frac{1}{2}$; 2) $\frac{1}{3}$; 3) $\frac{1}{5}$; 4) $\frac{1}{7}$; 5) $\frac{1}{3}$; 6) $\frac{2}{3}$; 7) $\frac{1}{3}$; 8) $\frac{7}{9}$; 9) $\frac{3}{7}$; 10) $\frac{1}{2}$; 11) $\frac{1}{2}$; 12) $\frac{1}{6}$; 13) $\frac{1}{2}$; 14) $\frac{2}{7}$; 15) $\frac{5}{11}$; 16) $\frac{3}{5}$; 17) $\frac{2}{39}$; 18) $\frac{17}{39}$; 19) $\frac{7}{17}$; 20) $\frac{1}{10}$

E) QUIZ.

Rewrite the following fractions in lowest terms.

1) $\frac{3}{6}$

2) $\frac{14}{21}$

3) $\frac{6}{36}$

4) $\frac{26}{52}$

5) $\frac{42}{72}$

Answers to E “Quiz”: 1) $\frac{1}{2}$; 2) $\frac{2}{3}$; 3) $\frac{1}{6}$; 4) $\frac{1}{2}$; 5) $\frac{7}{12}$

III. Changing an Improper Fraction into a Mixed Number.

Introduction: This is the third of four parts on understanding fractions. You're going to look at some examples on how to change an improper fraction into a mixed number. Afterwards, you're going to try to do some problems on your own. There will be 20 problems for you to practice. After you're successful in doing the practice problems, try the short quiz. The answers can be found at the end of each section.

A) **Example 1:** Change $\frac{15}{7}$ into a mixed number.

Step 1: Rewrite the fraction as a division problem. The denominator becomes the divisor and the numerator becomes the dividend.

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor} \overline{) \text{Dividend}} \end{array}$$

$$7 \overline{) 15}$$

Step 2: Divide 7 into 15.

$$\begin{array}{r} 2 \\ 7 \overline{) 15} \\ \underline{-14} \quad (2 \times 7 = 14) \\ 1 \quad \longrightarrow \text{Remainder} \end{array}$$

Step 3: Use the different parts of the division problem to construct the mixed number.

The parts of a mixed number:

$$\text{Whole Number} \frac{\text{Numerator}}{\text{Denominator}}$$

The parts of the mixed number replaced with the parts of a division problem:

$$\text{Quotient} \frac{\text{Remainder}}{\text{Divisor}}$$

The quotient is 2; the remainder is 1; and the divisor is 7. Substituting these values in the proper place, the end result is a mixed number.

$$2 \frac{1}{7}$$

B) Example 2: Change $\frac{27}{6}$ into a mixed number.

Step 1: Rewrite the fraction as a division problem. The denominator becomes the divisor and the numerator becomes the dividend.

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor} \overline{) \text{Dividend}} \end{array}$$

$$6 \overline{) 27}$$

Step 2: Divide 6 into 27.

$$\begin{array}{r} 4 \\ 6 \overline{) 27} \\ \underline{-24} \quad (4 \times 6 = 24) \\ 3 \quad \longrightarrow \text{Remainder} \end{array}$$

Step 3: Use the different parts of the division problem to construct the mixed number.

The parts of a mixed number:

$$\text{Whole Number} \frac{\text{Numerator}}{\text{Denominator}}$$

The parts of the mixed number replaced with the parts of a division problem:

$$\text{Quotient} \frac{\text{Remainder}}{\text{Divisor}}$$

The quotient is 4; the remainder is 3; and the divisor is 6. Substituting these values in the proper place, the end result is a mixed number.

$$4 \frac{3}{6}$$

Step 4: Make sure the fraction part is in lowest terms.

The fraction part is

$$\frac{3}{6}$$

Need to reduce the numerator and denominator by the same value. The numerator and denominator can be divided by 3.

$$\frac{3 \div 3}{6 \div 3}$$

$$\frac{1}{2}$$

Step 5: Rewrite the reduced mixed number.

$$4 \frac{1}{2}$$

C) Example 3: Change $\frac{56}{8}$ into a mixed number.

Step 1: Rewrite the fraction as a division problem. The denominator becomes the divisor and the numerator becomes the dividend.

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor} \overline{) \text{Dividend}} \end{array}$$

$$8 \overline{) 56}$$

Step 2: Divide 8 into 56.

$$\begin{array}{r} 7 \\ 8 \overline{) 56} \\ \underline{-56} \quad (7 \times 8 = 56) \\ 0 \quad \longrightarrow \text{Remainder} \end{array}$$

Step 3: Use the different parts of the division problem to construct the mixed number.

The parts of a mixed number:

$$\text{Whole Number} \frac{\text{Numerator}}{\text{Denominator}}$$

The parts of the mixed number replaced with the parts of a division problem:

$$\text{Quotient} \frac{\text{Remainder}}{\text{Divisor}}$$

The quotient is 7; the remainder is 0; and the divisor is 8. Substituting these values in the proper place, the end result is a mixed number.

$$7 \frac{0}{8}$$

Rewrite the mixed fraction as an addition problem.

$$7 + \frac{0}{8}$$

Simplify the fraction part: $\frac{0}{8} = 0$

$$7 + 0$$

Do the addition.

$$7$$

Answer: 7

D) You try:

a) $\frac{19}{5}$

b) $\frac{12}{8}$

c) $\frac{12}{2}$

Answers to D "You Try":

a) $3 \frac{4}{5}$

b) $1 \frac{4}{8} = 1 \frac{1}{2}$

c) 6

E) PRACTICE PROBLEMS.

Change the following improper fractions into mixed numbers:

1) $\frac{17}{8}$

2) $\frac{11}{3}$

3) $\frac{36}{9}$

4) $\frac{47}{8}$

5) $\frac{25}{2}$

6) $\frac{55}{3}$

7) $\frac{32}{5}$

8) $\frac{13}{2}$

9) $\frac{45}{5}$

10) $\frac{10}{7}$

11) $\frac{38}{9}$

12) $\frac{20}{8}$

13) $\frac{22}{2}$

14) $\frac{41}{6}$

15) $\frac{46}{9}$

16) $\frac{48}{5}$

17) $\frac{57}{3}$

18) $\frac{14}{8}$

19) $\frac{21}{4}$

20) $\frac{49}{3}$

Answers to E "Practice Problems": 1) $2\frac{1}{8}$; 2) $3\frac{2}{3}$; 3) 4; 4) $5\frac{7}{8}$; 5) $12\frac{1}{2}$; 6) $18\frac{1}{3}$; 7) $6\frac{2}{5}$; 8) $6\frac{1}{2}$; 9) 9; 10) $1\frac{3}{7}$; 11) $4\frac{2}{9}$; 12) $2\frac{4}{8} = 2\frac{1}{2}$; 13) 11; 14) $6\frac{5}{6}$; 15) $5\frac{1}{9}$; 16) $9\frac{3}{5}$; 17) 19; 18) $1\frac{6}{8} = 1\frac{3}{4}$; 19) $5\frac{1}{4}$; 20) $16\frac{1}{3}$

F) QUIZ.

Change the following improper fractions into mixed numbers:

1) $\frac{35}{6}$

2) $\frac{12}{5}$

3) $\frac{18}{8}$

4) $\frac{29}{7}$

5) $\frac{15}{2}$

Answers to F “Quiz”: 1) $5\frac{5}{6}$; 2) $2\frac{2}{5}$; 3) $2\frac{2}{8} = 2\frac{1}{4}$; 4) $4\frac{1}{7}$; 5) $7\frac{1}{2}$

IV. Changing a Mixed Number Into an Improper Fraction

Taken from Treff, A. & Jacobs, D., Life Skills Mathematics, Media Materials, Inc. Baltimore, Maryland 1983, p 275.

Introduction: This is the fourth of four parts on understanding fractions. You're going to look at an example on how to change a mixed number into an improper fraction. Afterwards, you're going to try to do some problems on your own. There will be 20 problems for you to practice. After you're successful in doing the practice problems, try the short quiz. The answers can be found at the end of each section.

A) **Example:** Write the following fraction as an improper fraction: $3\frac{2}{5}$

REVIEW: Whole Number $\frac{\text{Numerator}}{\text{Denominator}}$

STEP 1: Multiply the whole number by the denominator.

$$3 \cdot 5 = 15$$

STEP 2: Add the numerator to the product from STEP 1.

$$2 + 15 = 17$$

STEP 3: Write the sum over the original denominator.

$$\frac{17}{5}$$

**FIRST RULE OF MULTIPLYING, DIVIDING, ADDING & SUBTRACTING FRACTIONS:
CHANGE ALL MIXED NUMBERS INTO IMPROPER FRACTIONS.**

B) **You Try:**

a) $2\frac{3}{5}$

b) $1\frac{1}{2}$

c) $3\frac{1}{4}$

Answers to B "You Try":

a) $\frac{13}{5}$

b) $\frac{3}{2}$

c) $\frac{13}{4}$

C) PRACTICE PROBLEMS.

Change the following mixed numbers into improper fractions:

1) $1\frac{2}{3}$

2) $1\frac{3}{4}$

3) $2\frac{1}{4}$

4) $1\frac{7}{8}$

5) $3\frac{2}{3}$

6) $6\frac{1}{2}$

7) $4\frac{2}{3}$

8) $5\frac{3}{4}$

9) $1\frac{1}{2}$

10) $1\frac{2}{5}$

11) $3\frac{1}{5}$

12) $4\frac{2}{7}$

13) $3\frac{6}{7}$

14) $4\frac{3}{5}$

15) $2\frac{5}{7}$

16) $1\frac{5}{6}$

17) $2\frac{3}{8}$

18) $2\frac{1}{3}$

19) $3\frac{4}{5}$

20) $4\frac{5}{6}$

Answers to "Practice Problems": 1) $\frac{5}{3}$; 2) $\frac{7}{4}$; 3) $\frac{9}{4}$; 4) $\frac{15}{8}$; 5) $\frac{11}{3}$; 6) $\frac{13}{2}$; 7) $\frac{14}{3}$; 8) $\frac{23}{4}$; 9) $\frac{3}{2}$; 10) $\frac{7}{5}$; 11) $\frac{16}{5}$; 12) $\frac{30}{7}$; 13) $\frac{27}{7}$; 14) $\frac{23}{5}$; 15) $\frac{19}{7}$; 16) $\frac{11}{6}$; 17) $\frac{19}{8}$; 18) $\frac{7}{3}$; 19) $\frac{19}{5}$; 20) $\frac{29}{6}$

D) QUIZ.

Change the following mixed numbers into improper fractions:

1) $1\frac{2}{3}$

2) $2\frac{3}{4}$

3) $3\frac{4}{5}$

4) $1\frac{1}{2}$

5) $2\frac{2}{3}$

Answers to D “Quiz”: 1) $\frac{5}{3}$; 2) $\frac{11}{4}$; 3) $\frac{19}{5}$; 4) $\frac{3}{2}$; 5) $\frac{8}{3}$